

روش نقطه ثابت

Simple Fixed-Point Iteration

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Simple Fixed-Point Iteration

- Rearrange the function so that x is on the left side of the equation:

$$f(x) = 0 \Rightarrow g(x) = x$$

$$x_{i+1} = g(x_i)$$

- Bracketing methods are “convergent”.
- Fixed-point methods may sometime “diverge”, depending on the starting point (initial guess) and how the function behaves.

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Simple Fixed-Point Iteration

Examples:

1.

$$f(x) = x^2 - x - 2 \quad x > 0$$

$$g(x) = x^2 - 2$$

2. $f(x) = \sin x \rightarrow x = g(x) = \sin x + x$

3. $f(x) = e^x - x \rightarrow x = g(x) = e^x$

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Simple Fixed-Point Iteration Convergence

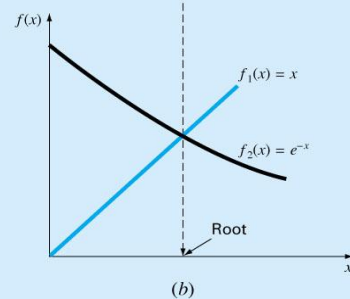
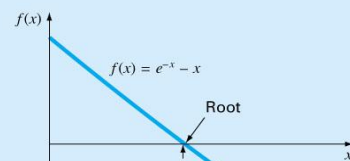
- $x = g(x)$ can be expressed as a pair of equations:

$$y_1 = x$$

$$y_2 = g(x) \dots$$

(component equations)

- Plot them separately.



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Simple Fixed-Point Iteration Convergence

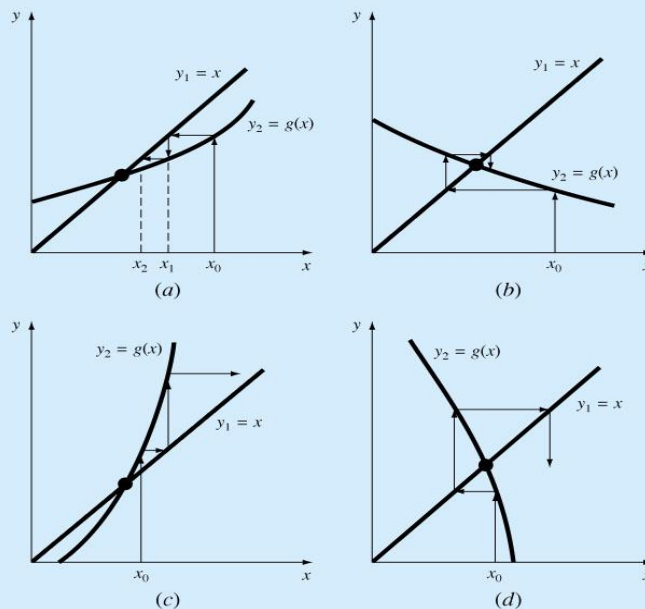
- Fixed-point iteration converges if :

$$|g'(x)| < 1 \quad (\text{slope of the line } f(x) = x)$$

- When the method converges, the error is roughly proportional to or less than the error of the previous step, therefore it is called "linearly convergent."

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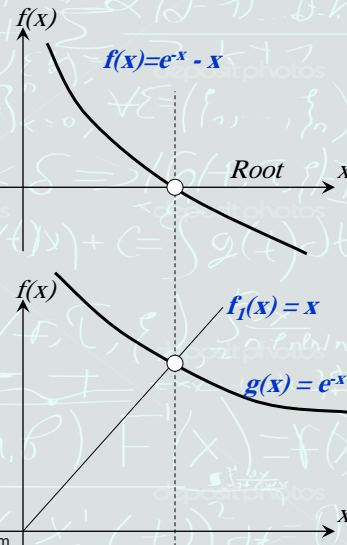
Simple Fixed-Point Iteration-Convergence



Example: Simple Fixed-Point Iteration

$$f(x) = e^x - x$$

1. $f(x)$ is manipulated so that we get $x=g(x) \rightarrow g(x) = e^{-x}$
2. Thus, the formula predicting the new value of x is: $x_{i+1} = e^{-x_i}$
3. Guess $x_0 = 0$
4. The iterations continues till the approx. error reaches a certain limiting value



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Example: Simple Fixed-Point Iteration

i	x_i	$g(x_i)$	$\epsilon_a\%$	$\epsilon_t\%$
0	0	1.0		
1	1.0	0.367879	100	76.3
2	0.367879	0.692201	171.8	35.1
3	0.692201	0.500473	46.9	22.1
4	0.500473	0.606244	38.3	11.8
5	0.606244	0.545396	17.4	6.89
6	0.545396	0.579612	11.2	3.83
7	0.579612	0.560115	5.90	2.2
8	0.560115	0.571143	3.48	1.24
9	0.571143	0.564879	1.93	0.705
10	0.564879		1.11	0.399

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Flow Chart – Fixed Point

