False-position Method

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- The bisection method divides the interval x_l to x_u in half not accounting for the magnitudes of $f(x_l)$ and $f(x_u)$. For example if $f(x_l)$ is closer to zero than $f(x_u)$, then it is more likely that the root will be closer to $f(x_l)$.
- False position method is an alternative approach where $f(x_l)$ and $f(x_u)$ are joined by a straight line; the intersection of which with the x-axis represents and improved estimate of the root.

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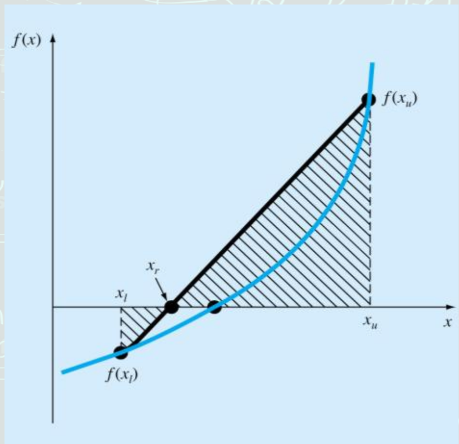

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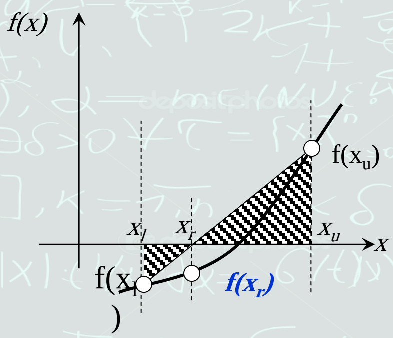
False-position Method -Procedure



$$\frac{f(x_l)}{x_r - x_l} = \frac{f(x_u)}{x_r - x_u}$$

$$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$$

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False-position Method -Procedure

Step 1: Choose lower x_l and upper x_u guesses for the root such that: $f(x_l).f(x_u)<0$

Step 2: The root estimate is:

$$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$$

Step 3: Subdivide the interval according to:

- If $(f(x_l).f(x_r)<0)$ the root lies in the lower subinterval; $x_u = x_r$ and go to step 2.
- If $(f(x_l).f(x_r)>0)$ the root lies in the upper subinterval; $x_l = x_r$ and go to step 2.
- If $(f(x_l).f(x_r)=0)$ the root is x_r and stop

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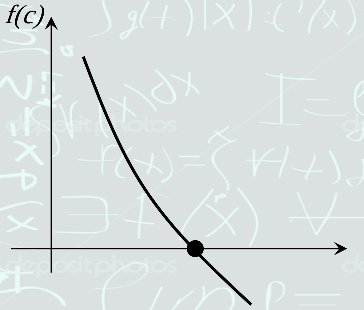
False position method: Example

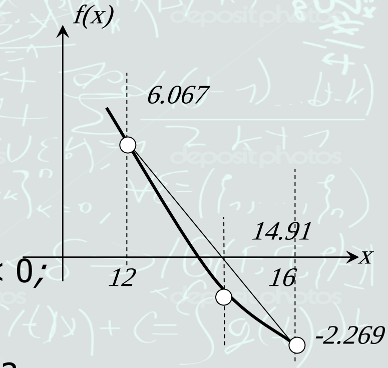
- The parachutist velocity is $v = \frac{mg}{c}(1 - e^{-\frac{c}{m}t})$
- What is the drag coefficient c needed to reach a velocity of 40 m/s if m =68.1 kg, t =10 s, g= 9.8 m/s²



$$f(c) = \frac{mg}{c}(1 - e^{-\frac{c}{m}t}) - v$$

$$f(c) = \frac{667.38}{c}(1 - e^{-0.146843c}) - 40$$

False position method: Example



1. Assume x_l = 12 and x_u=16

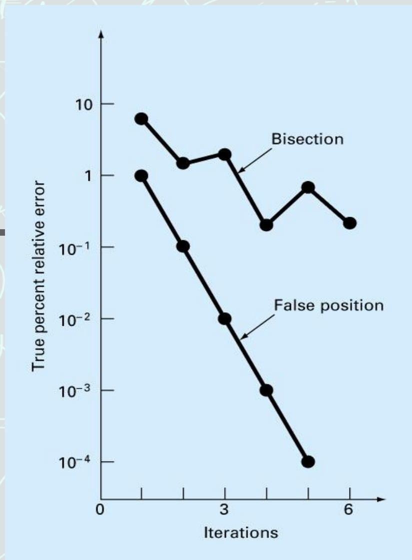
   $f(x_l)$= 6.067 *and* $f(x_u)$= -2.269

2. The root: x_r=*14.9113*

   f(12) . *f*(14.9113) = -1.5426 < 0;

3. The root lies bet. 12 and 14.9113.
4. Assume x_l = 12 and x_u=14.9113, $f(x_l)$=*6.067* *and* $f(x_u)$=*-0.2543*
5. The new root x_r= *14.7942*
6. This has an approximate error of 0.79%

False position method: Example



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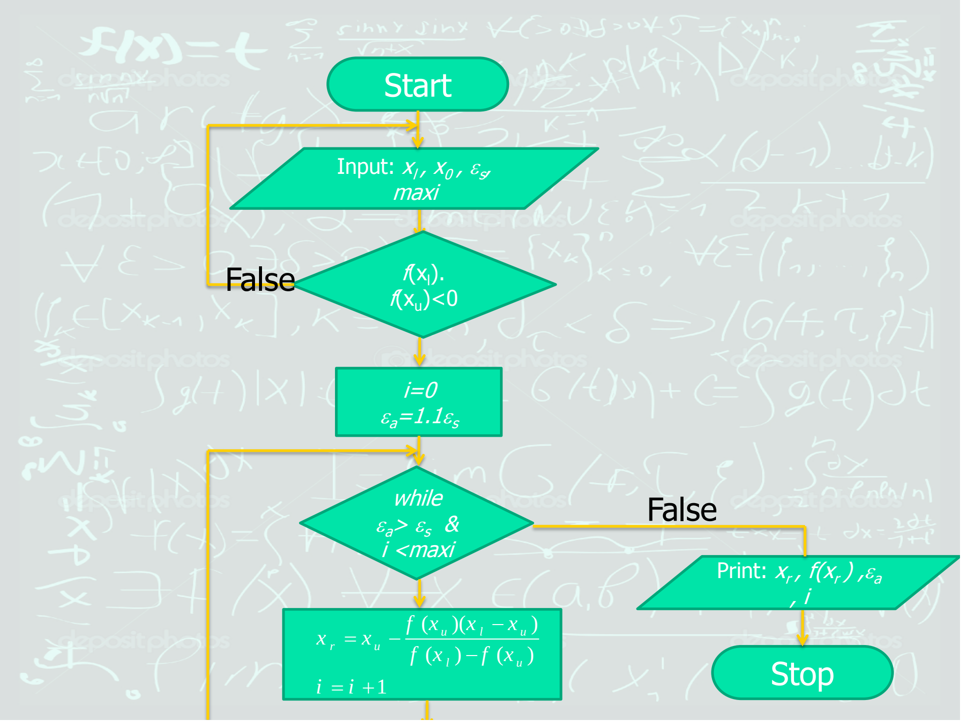
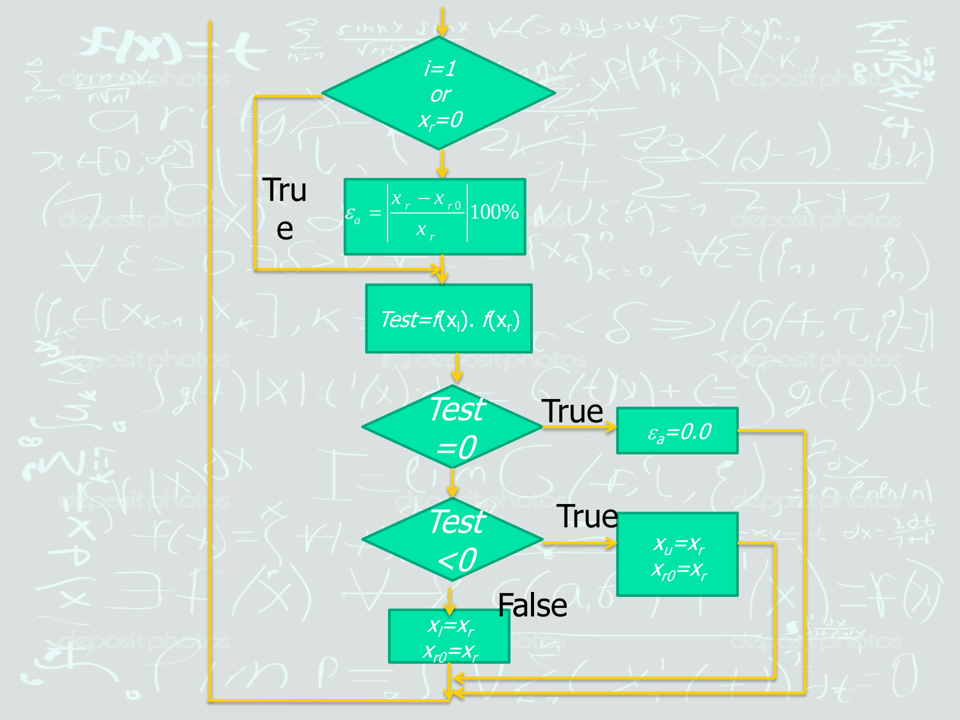

Start
Input: x_l, x_0, ε_s, maxi
False
$f(x_l).f(x_u)<0$
$i=0$
$\varepsilon_a=1.1\varepsilon_s$
while $\varepsilon_a>\varepsilon_s$ & $i<maxi$
False
Print: x_r, $f(x_r)$, ε_a, i
$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$
$i = i + 1$
Stop


$i=1$ or $x_r=0$
Tru e
$\varepsilon_a = \left|\frac{x_r - x_{r0}}{x_r}\right| 100\%$
$Test=f(x_l).f(x_r)$
Test =0
True
$\varepsilon_a=0.0$
Test <0
True
$x_u=x_r$
$x_{r0}=x_r$
False
$x_l=x_r$
$x_{r0}=x_r$

False Position Method-Example 2

A Case Where Bisection Is Preferable to False Position

Problem Statement. Use bisection and false position to locate the root of

$$f(x) = x^{10} - 1$$

between $x = 0$ and 1.3.

Solution. Using bisection, the results can be summarized as

Iteration	x_l	x_u	x_r	ε_a (%)	ε_t (%)
1	0	1.3	0.65	100.0	35
2	0.65	1.3	0.975	33.3	2.5
3	0.975	1.3	1.1375	14.3	13.8
4	0.975	1.1375	1.05625	7.7	5.6
5	0.975	1.05625	1.015625	4.0	1.6

Thus, after five iterations, the true error is reduced to less than 2 percent. For false position, a very different outcome is obtained:

Iteration	x_l	x_u	x_r	ε_a (%)	ε_t (%)
1	0	1.3	0.09430		90.6
2	0.09430	1.3	0.18176	48.1	81.8
3	0.18176	1.3	0.26287	30.9	73.7
4	0.26287	1.3	0.33811	22.3	66.2
5	0.33811	1.3	0.40788	17.1	59.2

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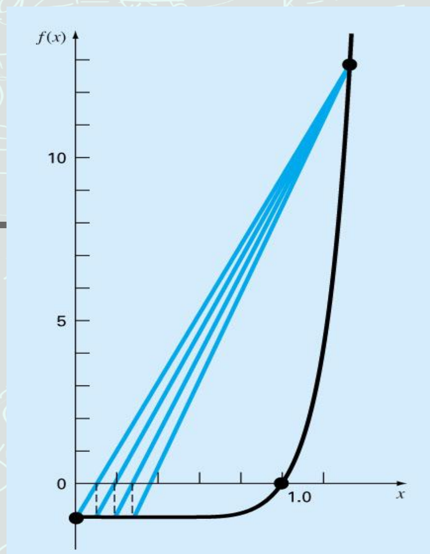
False Position Method - Example 2



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