Modeling the Feeding Process in Gill-Box Using Non-Linear Mass-Spring System

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Abstract—The purpose of this study is to investigate the behavior of the sliver pick-up from can in gill-box machines during feeding process. A model is presented to predict the height changes of can plate during the feeding process. To this aim, a mass-spring model is introduced. The behavior of the can plate is considered as linear spring system and the sliver weight in the can is also noted as time depended mass value. The mass spring derived equations are solved using Three-order Straight Forward Expansion method. Result of can plate height predicting by the model is compared to some experimental data which are collected from a wool yarn mill. The meaningful 12 percent difference between proposed model and the experimental value proves the acceptable result of desired model that can be used for predicting the height of can plate by the aim of machine speed and sliver count.

Key words: Can, gill-box, mass-spring model, straight forward expansion

I. INTRODUCTION

Sliver evenness has strong influence on the yarn quality. One of the most important factors that affect on sliver evenness is machine adjustment and feeding process. In the earlier works, Taylor, Grosberg, Audivert and Vidiella, Sengupta and Kapoor and Cherif and his coworkers studied on the influence of delivery speed on the fiber behavior in the drafting zone to explain the sliver quality parameters [1-5]. Ernst et al. as well as Cherif and Wulfhorst observed that at high draw frame speed, the degree of fiber parallelization obtained in the draw frame decreases [6-7]. Ishtiaque et al. investigated draw frame delivery speed, card machine draft and card machine coiler diameter at card have strong influence on the fiber orientation parameters of the sliver [8].

Sliver feeding parameters effect on the quality of sliver as well as draw frame machine adjustment. Sliver feeding parameters can be defined as:
- Distance between can and draw frame,
- Can edge smoothness, and
- Can plate height during sliver feeding process.

All of these parameters effect on feeding sliver quality and the evenness of produced sliver. Two first parameters can be easily checked and adjusted while the distance between sliver in the can and machine feeding zone should be fixed when can is full till empty. For this purpose, can plate position should be changed according to machine feeding speed and sliver count. To achieve this goal, the can spring stiffness should be determined.

In this paper a model base of mass spring system is introduced to predict can plate position by considering the can spring stiffness, machine speed in feeding zone and the sliver count. The suggested model can be applied to select can spring stiffness in yarn manufactory.

II. MODELING

In order to simulate the behavior of the sliver pick-up from can in gill-box machines, a model consists of a mass and a linear spring was introduced in a manner that the can plate behavior was supposed as linear spring system and the amount of sliver in can in feeding process considered as a time-variable mass system. The schematic of proposed model is illustrated in Figure 1.

![Fig. 1. a) The proposed model for predicting of height changes of can plate during feeding process of sliver from can. b) Can structure](image-url)

A. Assumptions

In order to simplify the model, some assumptions were made:
- The spring behavior was assumed to be linear,
- Mass was expected to be centered, and
- The removal of sliver from can was supposed to be uniform.

B. Model analysis

As mentioned the model is consisted of a mass and a linear spring. Equation 1 shows the employed equation of motion.

\[ M \ddot{X} + KX = Mg \]  

where, \( M \) is the mass of can plate with amount of sliver in above it, \( \ddot{X} \) shows the displacement of can plate, \( X \) demonstrates the acceleration of can plate, \( g \) is gravity acceleration and \( K \) shows the stiffness coefficient of
spring. It should be noted that the amount of sliver is reduced by feeding it to the gill-box machine. So \( M \) is a time variable parameter and can be written as:

\[
M = m - \dot{m}(t)
\]  
(2)

where \( m \) shows the mass of can plate and \( \dot{m}(t) \) demonstrates the mass of sliver which is time dependent and can be defined by Equation 3.

\[
\dot{m}(t) = a \times t
\]  
(3)

where \( a \) is depend on speed machine and can be computed from Equation 4 while \( t \) denotes time.

\[
a = \text{Sliver density} \times \text{Feeding speed} / 60 \times 1000
\]  
(4)

By considering

\[
\bar{x} + x = X
\]  
(5)

where \( X \) is the initial static equilibrium position and \( x \) is position of can plate considering the initial static equilibrium position, as is shown in Figure 2

\[
kx = mg
\]  
(6)

Fig. 2. Mass center position.

Equation (7) can be obtained by substituting Equation (5) into Equation (1).

\[
M \ddot{x} + K(x + \bar{x}) = Mg
\]  
(7)

Then, Equation (8) can be derived by substituting Equations (2) and (3) into Equation (7).

\[
(m - a \times t) \ddot{x} + K(x + \bar{x}) = (m - a \times t) g
\]  
(8)

Later, Equation (9) can be obtained by dividing the two sides of Equation (8) by \( m \).

\[
\left(1 - \frac{a}{m}\right) \ddot{x} + \frac{K}{m}(x + \bar{x}) = \left(1 - \frac{a}{m}\right) g
\]  
(9)

Finally, by substituting Equation (6) into Equation (9), Equation (10) could be derived.

\[
\left(1 - \frac{a}{m}\right) \ddot{x} + \frac{K}{m} x = -\frac{a}{m} g
\]  
(10)

For simplification, new parameters (\( \alpha \) and \( \gamma \)) are introduced as follows:

\[
\alpha = \frac{a}{m}
\]  
(11)

\[
\gamma = \frac{a}{m} g
\]  
(12)

Besides, \( \omega_n \) is considered as natural frequency of the linear system, as shown by Equation (13).

\[
\omega_n^2 = \frac{K}{m}
\]  
(13)

By considering Equation (12), Equation (9) can be written:

\[
(1 - \alpha t) \ddot{x} + \omega_n^2 x = -\gamma
gt\]

Straight Forward Expansion method was then used to solve Equation (14) [9]. To solve this equation, new parameters, i.e. \( \beta \) and \( \lambda \), could be defined as:

\[
\beta = \frac{\alpha}{\epsilon^2}
\]  
(15)

\[
\lambda = \frac{\gamma}{\epsilon^2}
\]  
(16)

Equation (17) can be obtained by substituting Equations (15) and (16) into Equation (14).

\[
(1 - \epsilon^2 \beta t) \ddot{x} + \omega_n^2 x = -\lambda \epsilon^2 t
\]  
(17)

where, \( \epsilon \) is perturbation parameter.

The solution is assumed to be in the form of an infinite series of the perturbation parameter \( \epsilon \) which could be represented as Equation (18).

\[
x(t) = \epsilon x_1(t) + \epsilon^2 x_2(t) + \epsilon^3 x_3(t) + ... 
\]  
(18)

In this study, the third-order perturbation method was used to solve the differential equation, therefore the response of vibration could be considered as:

\[
x(t) = \epsilon x_1(t) + \epsilon^2 x_2(t) + \epsilon^3 x_3(t)
\]  
(19)

Then, Equation (20) can be obtained by substituting Equation (19) into Equation (17).

\[
(1 - \epsilon^2 \beta t) \ddot{x}_1 + \omega_n^2 x_1 = -\lambda \epsilon^2 t
\]  
(20)

Since the perturbation parameter \( \epsilon \) could have been chosen arbitrarily, the coefficients of the various powers of \( \epsilon \) must be equated to zero. This leads to a system of equations which can be solved successively:

\[
\epsilon^1: x_1 + \omega_n^2 x_1 = 0
\]  
(21)

\[
\epsilon^2: x_2 + \omega_n^2 x_2 = -\lambda t
\]  
(22)

\[
\epsilon^3: x_3 + \omega_n^2 x_3 = +\beta \epsilon t
\]  
(23)

To solve Equation (21), \( x_1 \) is considered as follow:

\[
x_1 = C \sin(\omega_n t + \varphi_0)
\]  
(24)

The coefficients \( C \) and \( \varphi_0 \) can be calculated by the initial conditions \( x_0(t) = 0 \) and \( x_0'(t) = 0 \). To solve Equation (22), \( x_0 \) could be considered as follow:

\[
x_2 = -\frac{\lambda t}{\omega_n^2}
\]  
(25)

Then, Equation (26) can be obtained by substituting Equation (24) into Equation (23).

\[
x_3 = -\frac{C \omega_n \beta}{2} \cos(\omega_n t + \varphi_0)
\]  
(26)

By substituting Equations (24) to (26) into Equation (19), the response of the system could be obtained as follows:

\[
x = \epsilon C \sin(\omega_n t + \varphi_0) - \frac{\lambda \epsilon^2}{\omega_n^2} t - C \epsilon^3 \omega_n \beta \cos(\omega_n t + \varphi_0)
\]  
(27)

Equation (28) can be obtained by considering \( \epsilon C = \lambda \).
\[ x = A \sin(\omega_0 t + \phi_0) - \frac{g}{\omega_0^2} t + \frac{A\omega_0}{\omega_0^2} \cos(\omega_0 t + \phi_0) \quad (28) \]

Now, to solve Equation (28) the initial conditions \( x_0(t) = 0 \) and \( x'(0) = 0 \) were supposed.

Equation (28) shows the height of can plate during sliver pick-up from can toward the equilibrium condition. By using the suggested model, the behavior of sliver pick-up of can could be predicated.

### III. RESULT AND DISCUSSION

**A. Study of model accuracy**

In order to investigate the model precision in comparison with experimental result, four gill-box machines in a domestic worsted mill have been selected. Table I illustrates feeding speed, average of sliver density and sliver length in can for each machine.

<table>
<thead>
<tr>
<th>Gill-box No.</th>
<th>Feeding speed (m/min)</th>
<th>Sliver density average (gram/meter)</th>
<th>Sliver length in can (meter)</th>
<th>Sliver weight in can (length x gram/meter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gill-box 1</td>
<td>15</td>
<td>22</td>
<td>1200</td>
<td>26.4</td>
</tr>
<tr>
<td>Gill-box 2</td>
<td>25</td>
<td>21</td>
<td>1210</td>
<td>25.2</td>
</tr>
<tr>
<td>Gill-box 3</td>
<td>23</td>
<td>21</td>
<td>1200</td>
<td>20</td>
</tr>
<tr>
<td>Gill-box 4</td>
<td>20</td>
<td>5</td>
<td>2500</td>
<td>12.5</td>
</tr>
</tbody>
</table>

Model parameters are shown in Table II. The parameters were extracted from the data illustrated in Table I.

<table>
<thead>
<tr>
<th>Gill-box No.</th>
<th>Initial mass (Kg)</th>
<th>Sliver weight in can plate (Kg)</th>
<th>Spring stiffness coefficient (K) (N/m²)</th>
<th>Mass change coefficient (a) (Kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gill-box 1</td>
<td>29.4</td>
<td>15</td>
<td>0.084</td>
<td>0.004</td>
</tr>
<tr>
<td>Gill-box 2</td>
<td>29.2</td>
<td>15</td>
<td>0.087</td>
<td>0.004</td>
</tr>
<tr>
<td>Gill-box 3</td>
<td>23</td>
<td>15</td>
<td>0.039</td>
<td>0.007</td>
</tr>
<tr>
<td>Gill-box 4</td>
<td>15.5</td>
<td>15</td>
<td>0.017</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Spring elongation of can was measured under a constant force to determine spring stiffness of each can. Then spring stiffness coefficient was calculated by considering Equation (29).

\[ F = K \times \Delta X \quad (29) \]

In Equation (29) \( K \) is spring stiffness coefficient (Newton/meter), \( F \) is force (Newton) and \( \Delta X \) is elongation (meter).

In order to study of model accuracy, height of can plate during sliver pick-up from can for each Gill-box were measured. Table 3 shows the comparison between result of model and experimental.

The difference between model prediction and the experimental measurements are shown in Table III, while the error of model is illustrated in this table too. Differences can be due to following reasons:

i- The spring behavior was assumed to be linear in model.
ii- Mass was thought to be centered, and
iii- Some types of errors occurred during experimental Measurement.

**B. Can spring stiffness determining**

As it mentioned earlier, distance between sliver in the can and machine feeding zone is one of the important parameters on sliver quality. So, the pick-up height should be constant. To have a constant pick-up height, the can plate must rise as same height as sliver decreases. Therefore, can plate position should be change according to machine feeding speed and sliver count. Hence, the can plate changes should depend on the spring stiffness. For this reason, the optimal value of spring stiffness was determined by the suggested model in this research.

| Machine No. | Time (minute) | Can plate height (result of model) (cm) | Can plate height (experimental) (cm) | Error (%)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gill-box 1</td>
<td>1</td>
<td>3.3</td>
<td>2.7</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6.7</td>
<td>6.8</td>
<td>15</td>
</tr>
<tr>
<td>Gill-box 2</td>
<td>3</td>
<td>10.08</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>13.45</td>
<td>12.5</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>17.1</td>
<td>16.1</td>
<td>6.2</td>
</tr>
<tr>
<td>Gill-box 3</td>
<td>1</td>
<td>3.48</td>
<td>3.1</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6.96</td>
<td>6.8</td>
<td>8.7</td>
</tr>
<tr>
<td>Gill-box 4</td>
<td>3</td>
<td>10.44</td>
<td>9.2</td>
<td>13.4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>13.93</td>
<td>12.9</td>
<td>7.9</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>17.4</td>
<td>15.8</td>
<td>10.1</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>3.12</td>
<td>3.7</td>
<td>10.2</td>
</tr>
<tr>
<td>Gill-box 5</td>
<td>7</td>
<td>6.24</td>
<td>6.6</td>
<td>11.4</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>9.37</td>
<td>8.6</td>
<td>8.9</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>11.6</td>
<td>10.7</td>
<td>8.4</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>13.6</td>
<td>12.2</td>
<td>13.3</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>2.72</td>
<td>2.4</td>
<td>13.3</td>
</tr>
<tr>
<td>Gill-box 6</td>
<td>12</td>
<td>4.08</td>
<td>3.7</td>
<td>10.2</td>
</tr>
</tbody>
</table>

![Simulated can plate height change](image)

**Fig. 3.** Comparison between time responses of the presented model for can plate height changes of Gill-box 1 and sliver height change during the sliver pick-up from can.

| Machine No. | Model of machine | Time (minute) | Height of can plate (cm) | Sliver height (cm) | Error (%)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gill-box 1</td>
<td>GC15-Schlumber(2000)</td>
<td>1</td>
<td>3.3</td>
<td>3.84</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>6.7</td>
<td>7.68</td>
<td>13</td>
</tr>
<tr>
<td>Gill-box 2</td>
<td>GC15-Schlumber(2000)</td>
<td>3</td>
<td>10.08</td>
<td>11.52</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>13.45</td>
<td>15.36</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>17.1</td>
<td>19.2</td>
<td>10</td>
</tr>
<tr>
<td>Gill-box 3</td>
<td>GC15-Schlumber(2000)</td>
<td>1</td>
<td>3.48</td>
<td>3.84</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>6.96</td>
<td>7.68</td>
<td>9</td>
</tr>
<tr>
<td>Gill-box 4</td>
<td>GC15-Schlumber(2000)</td>
<td>3</td>
<td>4.08</td>
<td>4.96</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>6.24</td>
<td>7.68</td>
<td>13</td>
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<tr>
<td></td>
<td></td>
<td>5</td>
<td>9.37</td>
<td>12.2</td>
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<td></td>
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<td>11.6</td>
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<td>27</td>
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<td></td>
<td></td>
<td>7</td>
<td>13.6</td>
<td>18.6</td>
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<td></td>
<td></td>
<td>8</td>
<td>17.4</td>
<td>19.2</td>
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<td></td>
<td></td>
<td>9</td>
<td>2.72</td>
<td>3.33</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>6.8</td>
<td>8.19</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td>7.6</td>
<td>10</td>
<td>24</td>
</tr>
</tbody>
</table>
Height of can plate over the time can be calculated through Equation (28). For calculation of sliver height during time, it was assumed that the change of sliver height is linear. With this assumption, the change coefficient of sliver height could be determined by Equation 30.

Change coefficient of sliver height = \[
\frac{\text{Initial height of silver into can}}{\text{Feeding time}}
\]  

(30)

As it is illustrated in Table VI, by using optimized value of spring stiffness coefficient, the height of sliver in can becomes about same with height of can plate (pick-up height is constant). Therefore the amount of stretch and unevenness of sliver can be reduced by selecting the appropriate value for spring stiffness coefficient of can. It is no doubt that the computation should be done on two factors, i.e. machine specifications and sliver properties.

**IV. Conclusions**

In this study, sliver pick-up behavior of can was investigated. A model base of mass spring system was introduced to predict the can plate position by considering can spring stiffness, machine speed in feeding zone and sliver count. Model was analyzed for gill-box cans in a worsted mill to gain higher accuracy. Results of the model were compared with experimental data and a significant 12% difference was observed between can plate height predicted by the model and the experimental results. It was also observed that the sliver height in can is more than height of can plate (pick-up height is not constant) therefore, to have a constant pick-up operation the spring stiffness should be determined by considering machine speed and sliver properties. Spring stiffness was determined according to machine specifications and sliver properties for each gill-box. It was observed that by selecting optimal value of spring stiffness parameter (K) of each gill-box, the height of the can plate become as same as sliver.

**REFERENCES**


